

# DECORATED TREES

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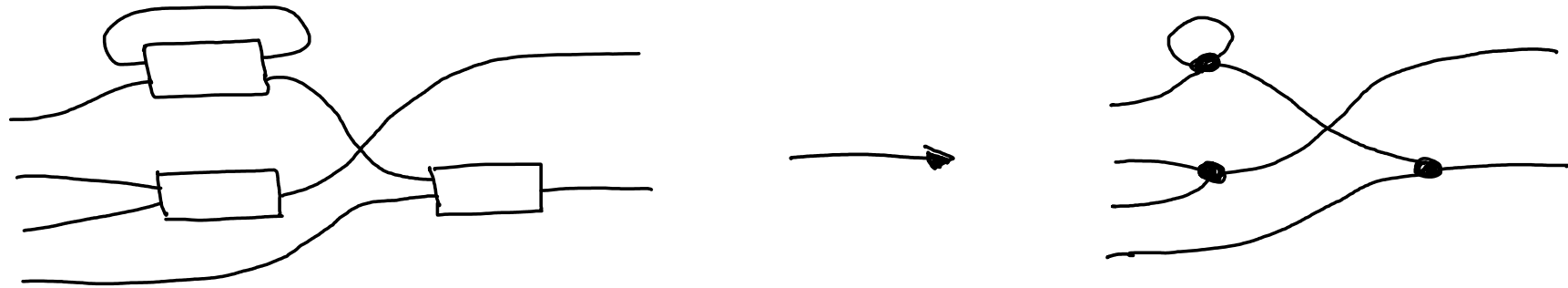
(jww Conor McBride)

SPLS

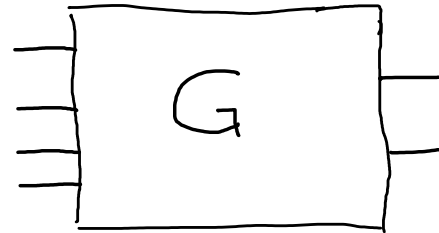
20 October  
2021

# GRAPHS MODEL CIRCUITS

wires  $\rightarrow$  edges, boxes  $\rightarrow$  vertices

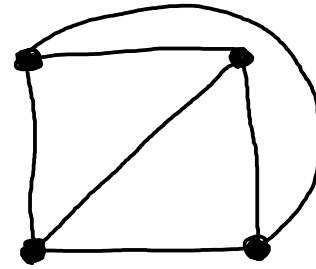
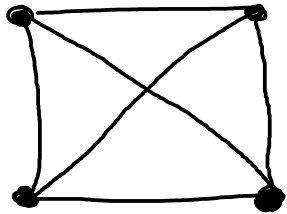


open graphs for  
representing  
inputs and outputs



# PLANAR GRAPHS

"No edges cross!"



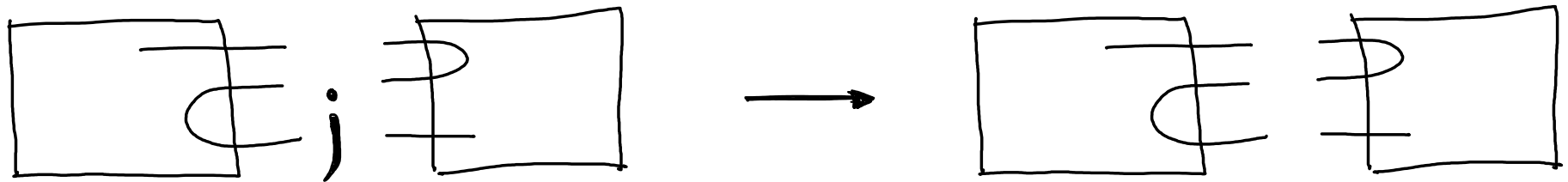
rotation is fixed

↘ order of edges around vertices

# A DATATYPE FOR GRAPHS

graphs are 2D  $\Rightarrow$  they're "unordered"

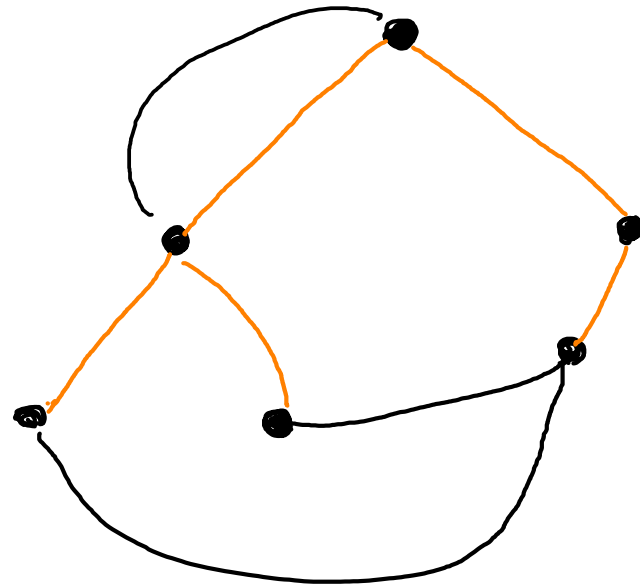
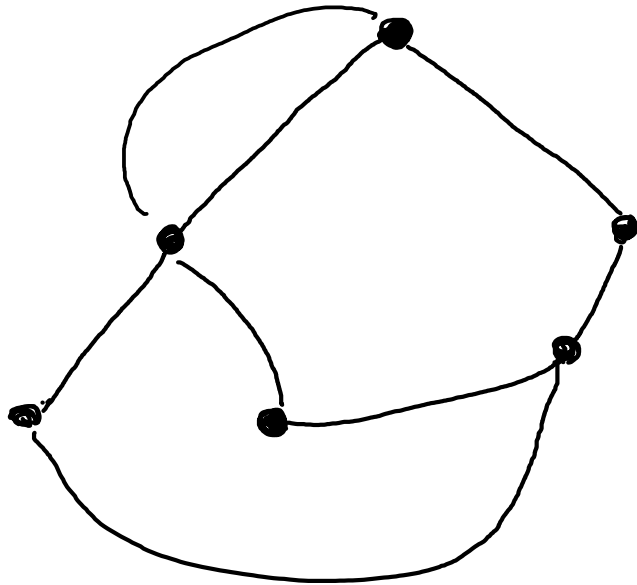
composition:



impose an order!

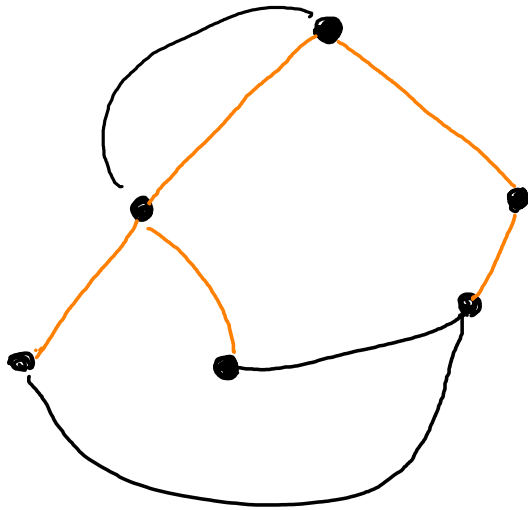
# SPANNING TREES

graph = spanning tree + extra edges



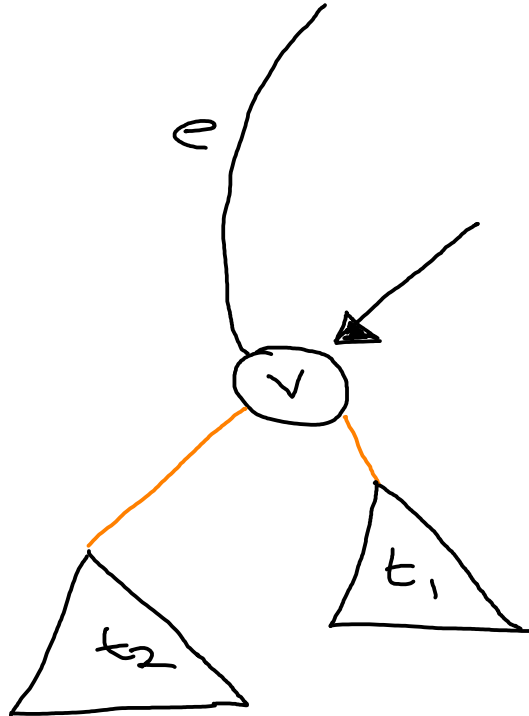
# PLANAR GRAPHS

how to organise the additional edges?



# THE DATATYPE

go round  
clockwise



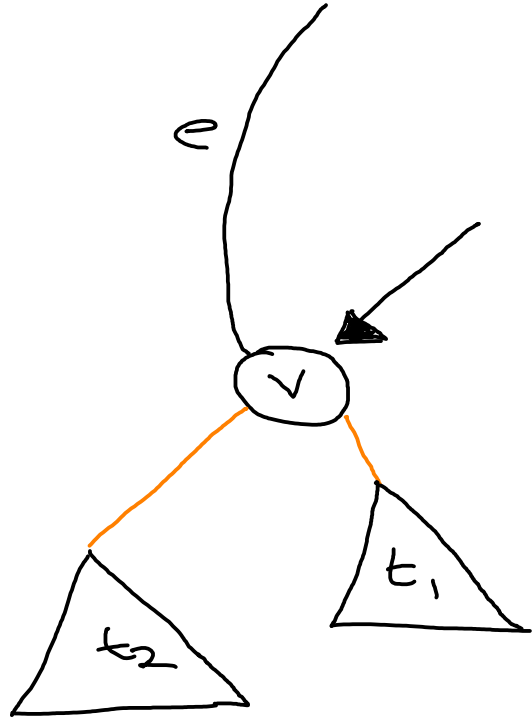
Stack to store cross edges

attached to a vertex:

- subtrees
- push edge
- pop edge

index by pre-stack  
and post-stack

# THE DATATYPE



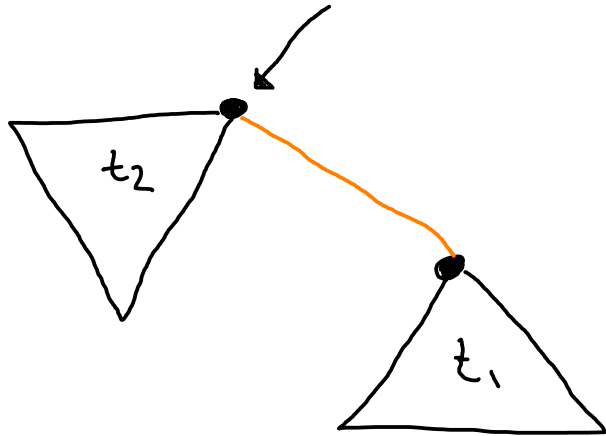
segments  
are places I can point to



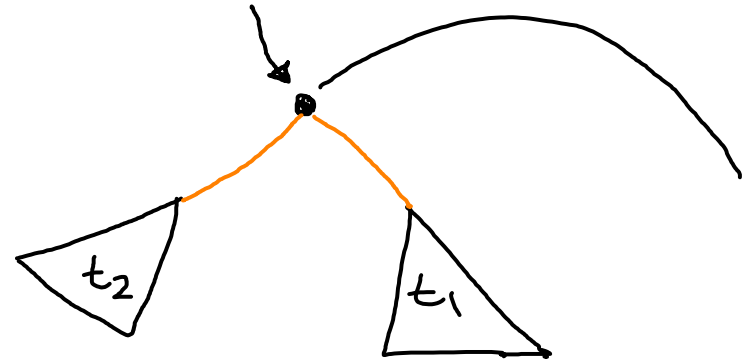
# MOVING THE ROOT

go round  
clockwise

move along the tree

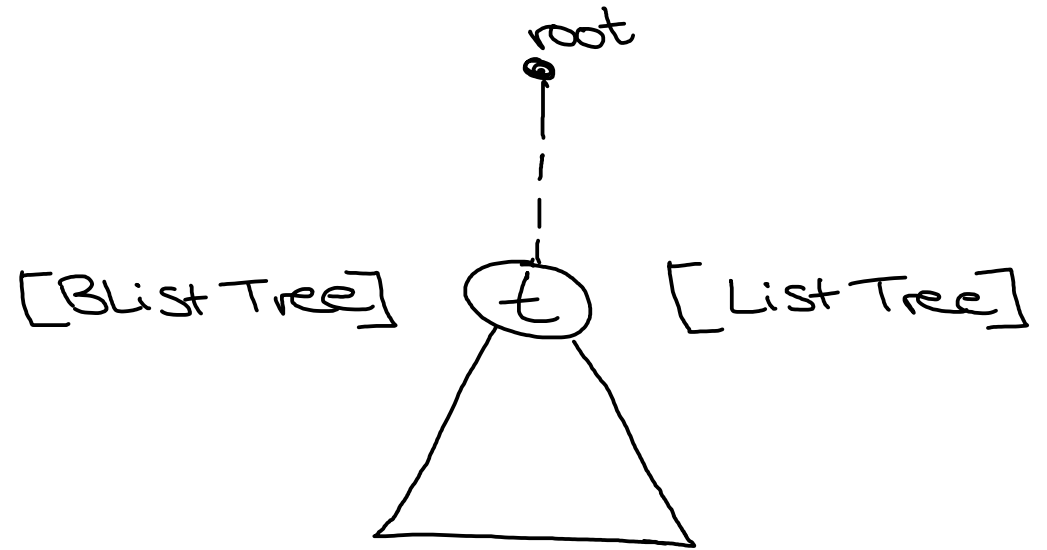
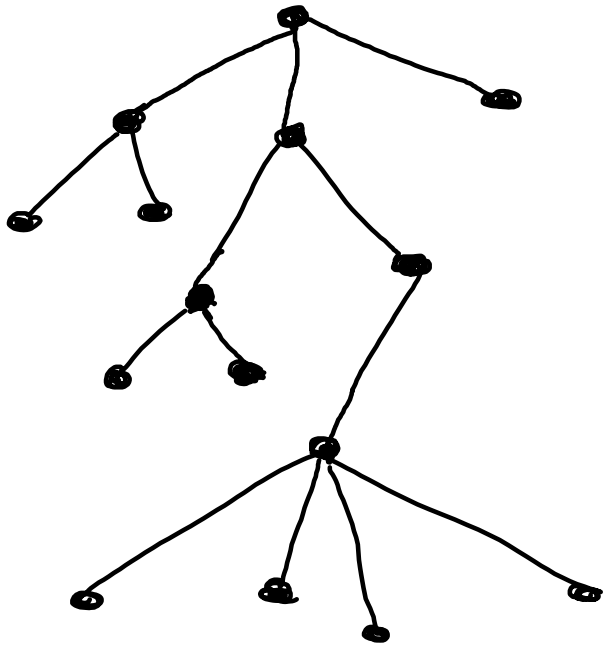


jump over a cross edge



# THE ZIPPER

zippers for trees

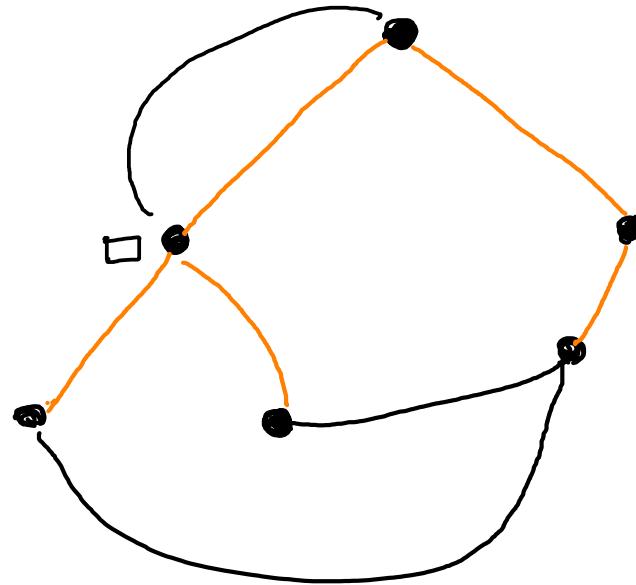
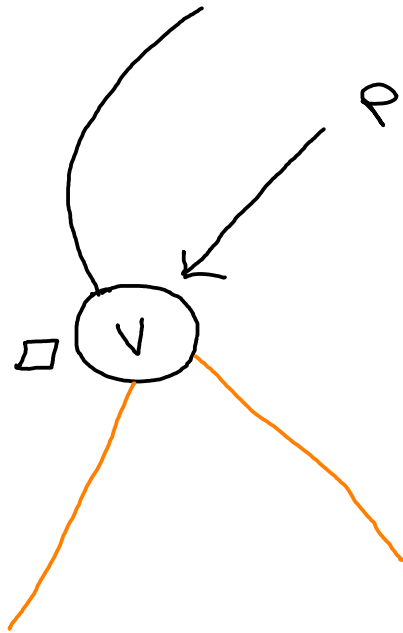


# THE ZIPPER

fixed  
spanning  
tree

go round  
clockwise

zipper for graphs



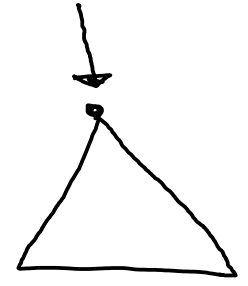
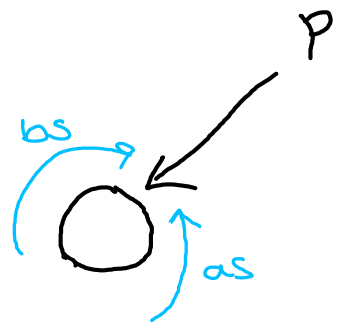
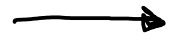
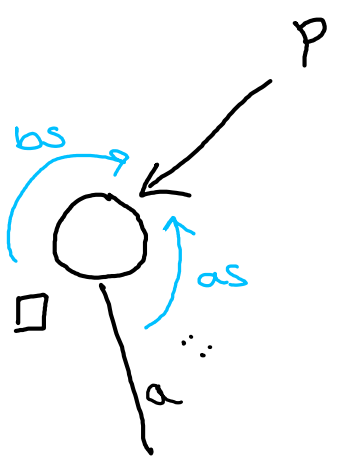
# THE ZIPPER

fixed spanning tree

go round clockwise

Zipper  $\rightarrow$  Tree

① back to the root



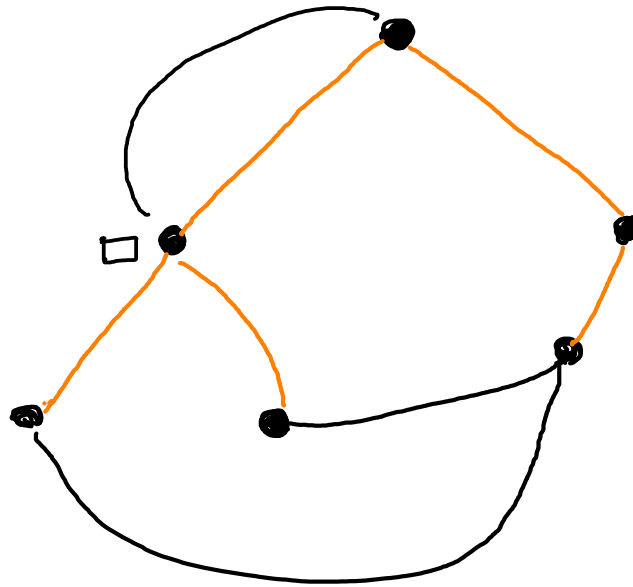
# THE ZIPPER

fixed  
spanning  
tree

go round  
clockwise

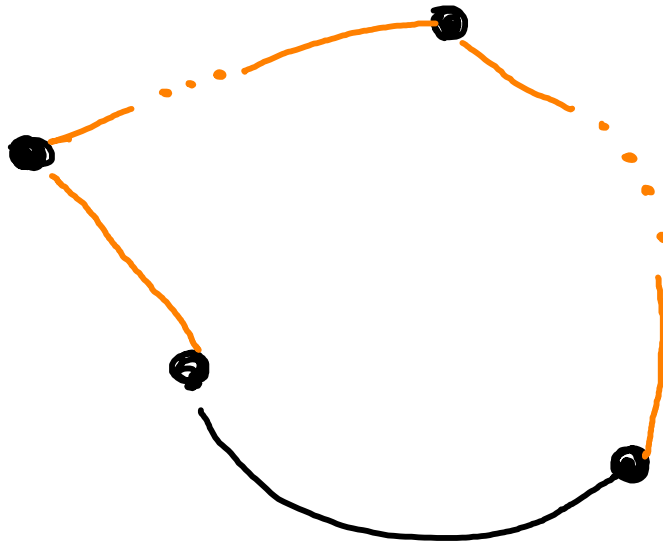
Zipper  $\rightarrow$  Tree

② root it here



# WHICH SPANNING TREE ?

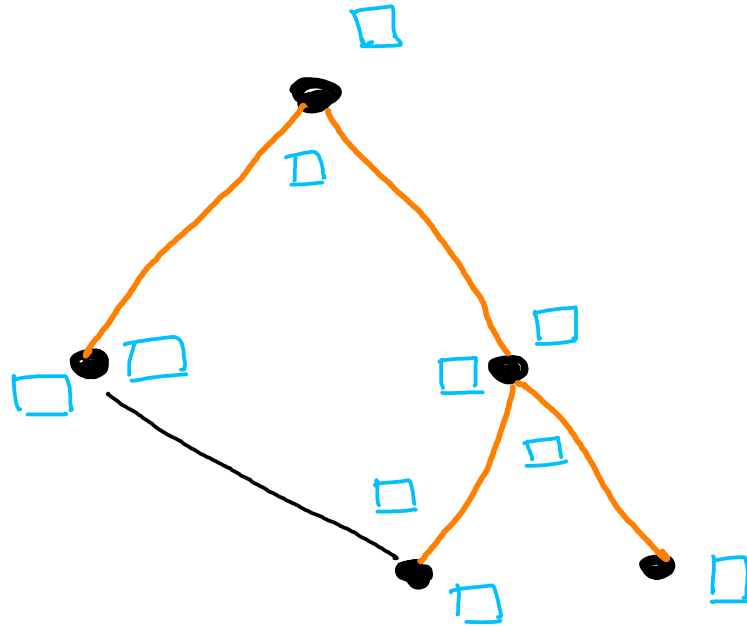
Changing the spanning tree is possible:



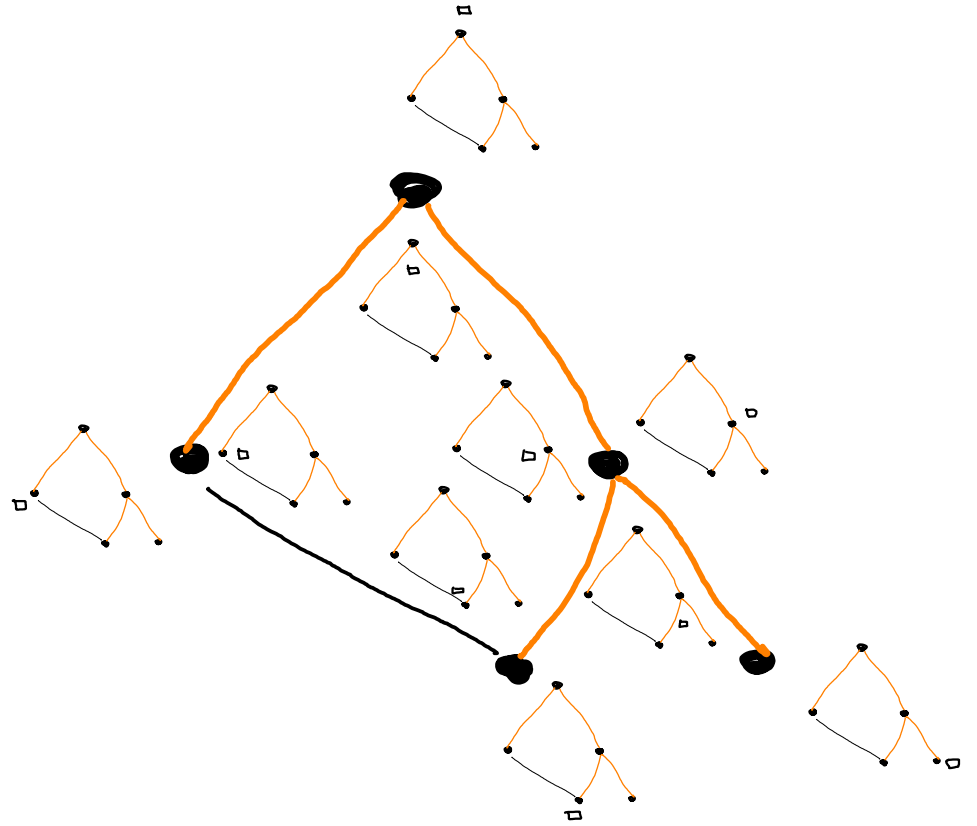
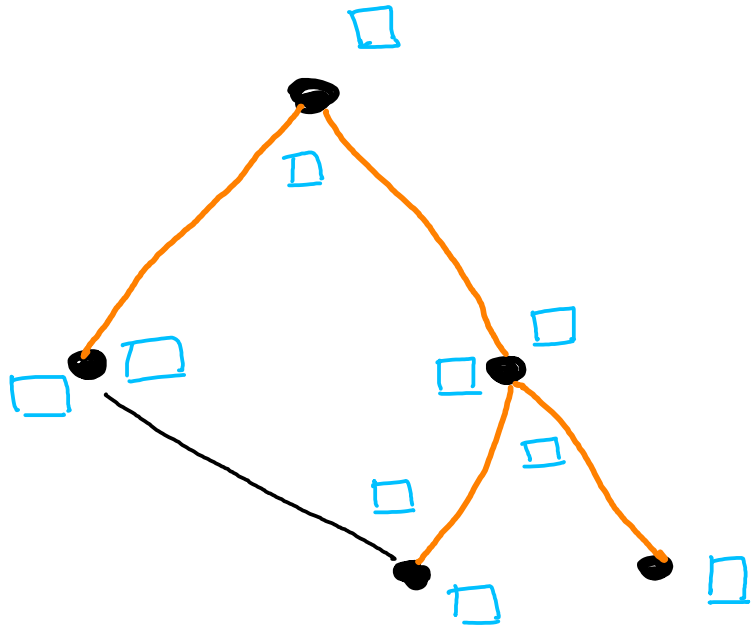
# MORE DECORATIONS!

Storing some data at the segment

Ⓐ Contexts



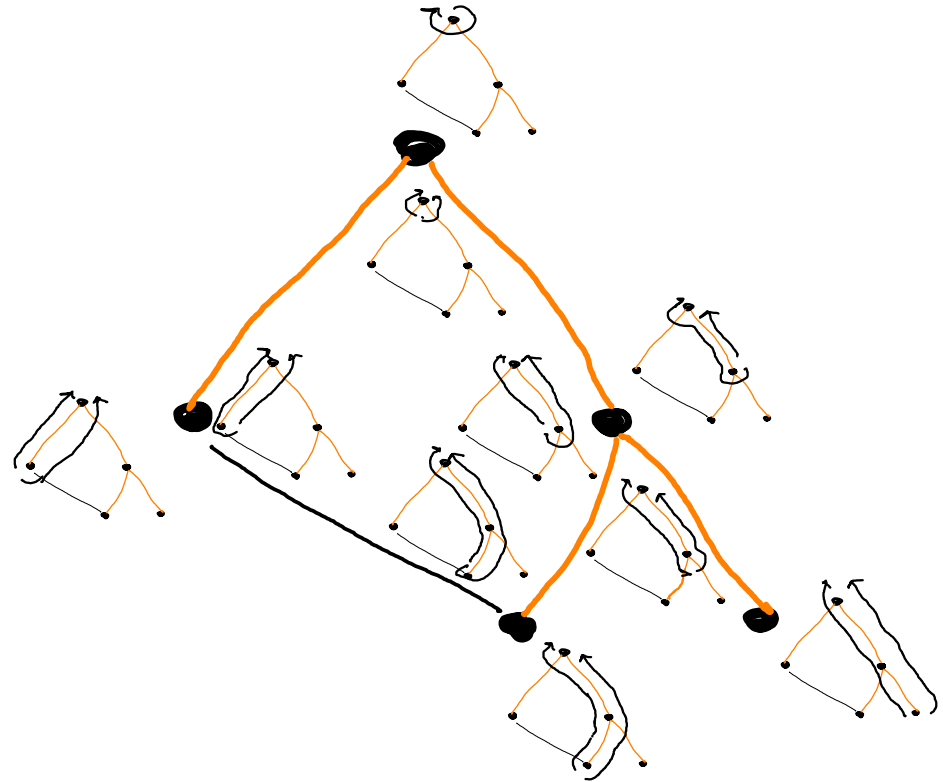
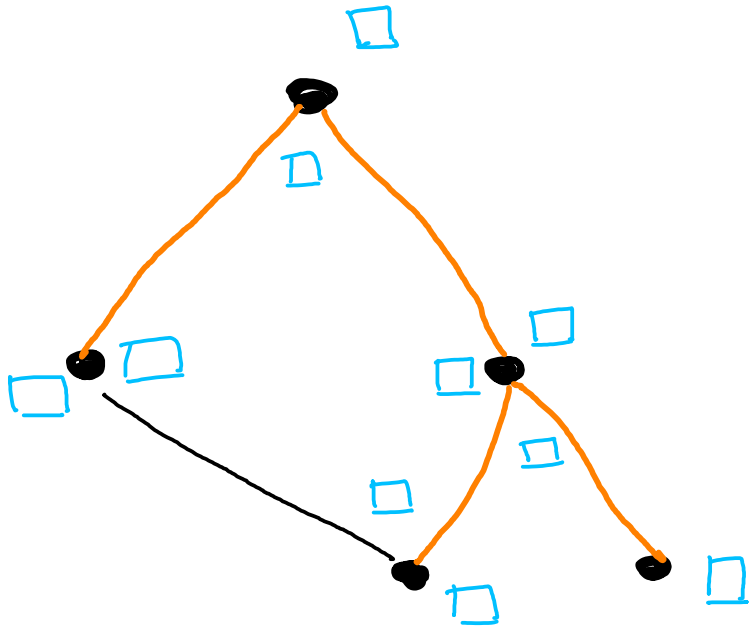
# MORE DECORATIONS!





# MORE DECORATIONS!

(B) the zipper at this position



(C) there's more options for decorations



# SUMMARY

graph = spanning tree + cross edges

stack to organise plane graph

navigating within a graph, zippers

decorate with contexts : it's a comonad

# SUMMARY

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decorate with contexts : it's a comonad

## THANK YOU FOR LISTENING

Get in touch with ideas or comments!

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# REFERENCES

The Zipper - Huet

∂ for Data, Differentiating Data Structures  
- Abbott, Altenkirch, Ghani, McBride

When is a Container a Comonad? - Ahman, Chapman, Uustalu

Comonadic Notions of Computation - Uustalu, Vene

