Control Flow as a Contour of Data Flow

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Workshop on Diagrammatic Methods
24 January 2023

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Data Flow:

- data dependency between programs
- information states
- track definition and usage of variables

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Control Flow

- order of execution of program fragments
- control flow analysis:
  important processing step for compiler optimisation, expose dead code fragments

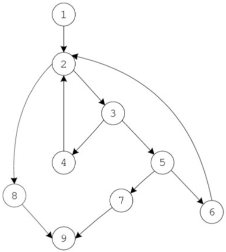
Control Flow Graphs

nodes: program blocks
edges: control flow

Source Program:

```
int binsearch(int x, int v[], int n)
{
   1 | int low, high, mid;
     | low = 0;
     | high = n - 1;
       while (low <= high) | 2
       {
          3 | mid = (low + high)/2;
            | if (x < v[mid])
                     high = mid - 1;  | 4
          5 | else if (x > v[mid])
                     low = mid + 1;   | 6
          7 | else return mid;
       }
       return -1; | 8
} | 9
```

CFG:



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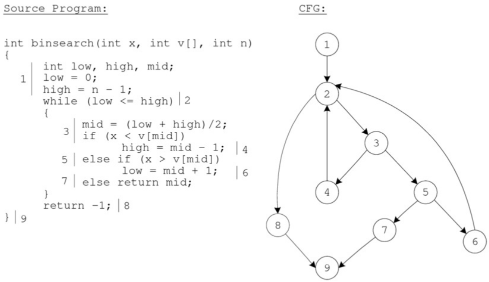
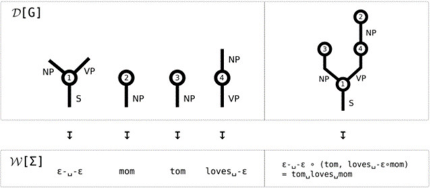


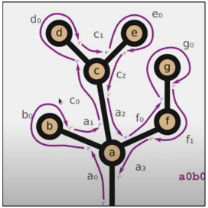
- disadvantage: graphs can be very big, algorithms on them are hard
- coming up: proposal for a different representation of control flow

Tree Contours[1]

- tree contours representing context-free languages
- trees represent derivations of a word
- linear contour around a derivation tree

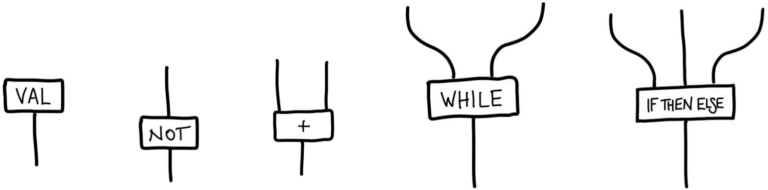
```
1 : S → NP VP
2 : NP → mom
3 : NP → tom
4 : VP → loves NP
```





[1] Melliès and Zeilberger, "Parsing as a lifting problem and the Chomsky-Schützenberger representation theorem".

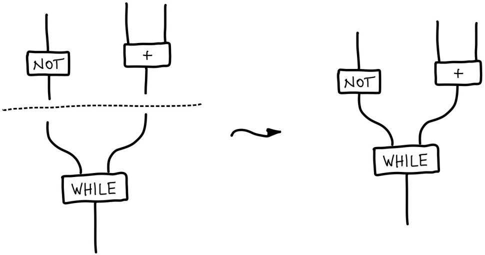
Abstract Syntax Generators



- imperative programs – variables, functions, control structures
- wires are variables
- multiple inputs, one output
- represented as a *species*: elements with multiple inputs and one output, e.g. *while* : $2 \to 1$

Composing Generators

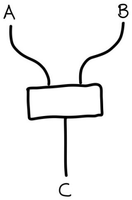
- composition is taking the free operad on the species
- amounts to building (partial) trees from the generators



Control Flow Contours(1)

given an operad, its generalised contour category consist of:

- objects: oriented colours of the operad



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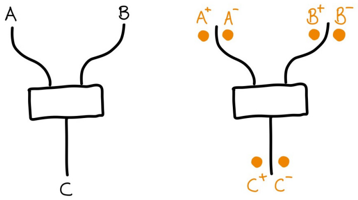

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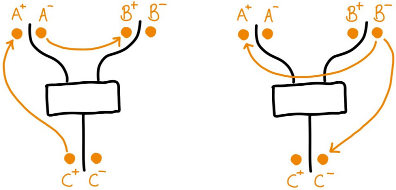
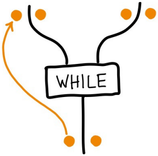


Control Flow Contours – Example 1

While loop:



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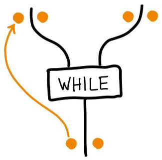

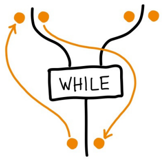

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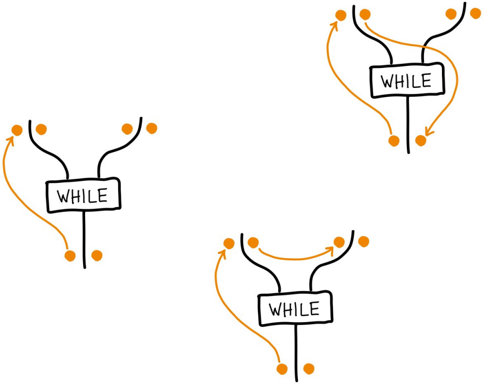

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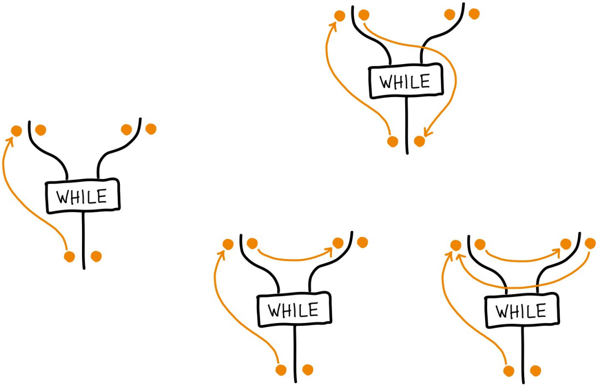

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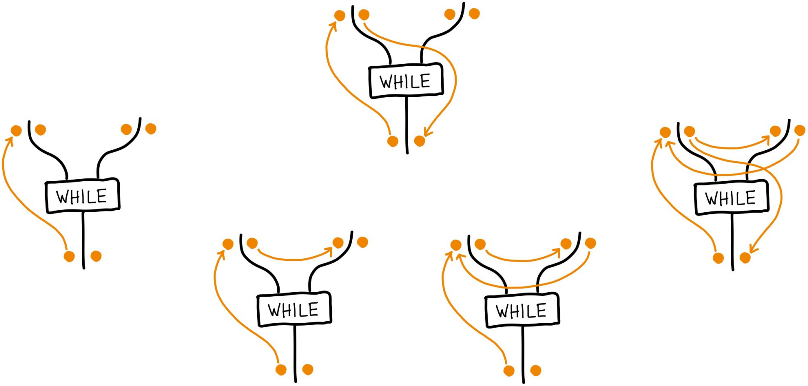
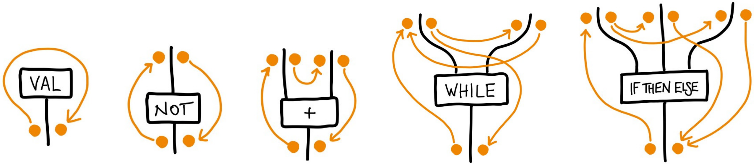
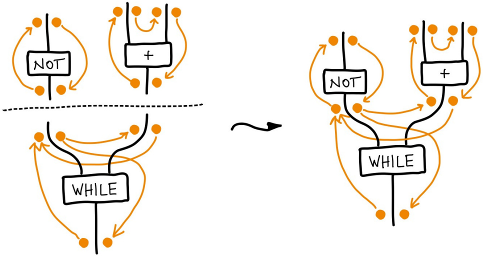


Control Flow Contours(2)

- edges represent control flow, for each of the generators
- contour is neither linear, nor ordered
- overapproximation of control flow



Contours compose!



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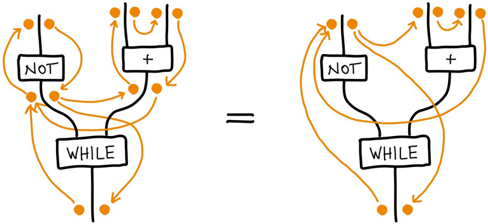

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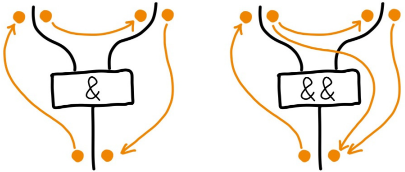

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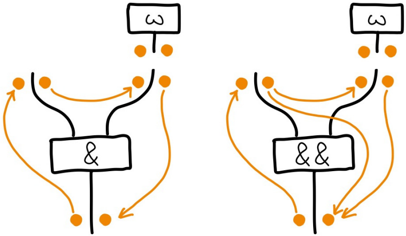

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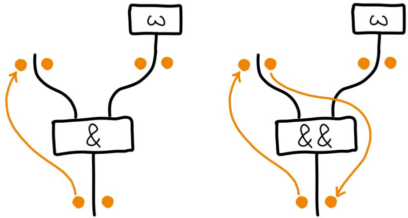


Control Flow Contours(3)

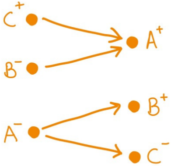
given an operad, its generalised contour category consist of:

- objects: oriented colours of the operad
- morphisms of the form $- \to +$, generated by colours and indices

This definition gives rise to a functor $\mathcal{C} : \mathbf{Operad} \to \mathbf{Cat}$.

Constructing Contours and back

Here's a functor $\mathcal{J}$: **Cat** $\to$ **Operad**:



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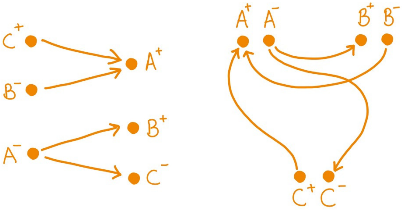

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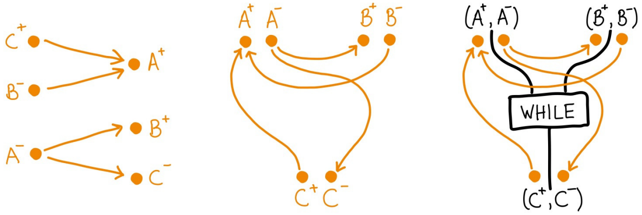

- colours are pairs of objects
- n-ary maps are sets of morphisms

Adjunction

- The contour is the left adjoint of the interior:
  $\mathbf{Operad}(\mathcal{O}, \mathcal{J}(\mathbf{C})) \cong \mathbf{Cat}(\mathcal{C}(\mathcal{O}), \mathbf{C})$

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$$\begin{tikzcd} O \arrow[d, "f"] & \mathcal{C}(O) \arrow[d, "g"] \\ \mathcal{J}(\mathbf{C}) & \mathbf{C} \end{tikzcd}$$

  for every map $f : O \to \mathcal{J}(\mathbf{C})$, there exists an object $D' \in \mathbf{Cat}$, a map $g : D' \to \mathbf{C}$ and a map $f_0 : O \to \mathcal{J} D'$, such that $f = \mathcal{J}(g) f_0$.

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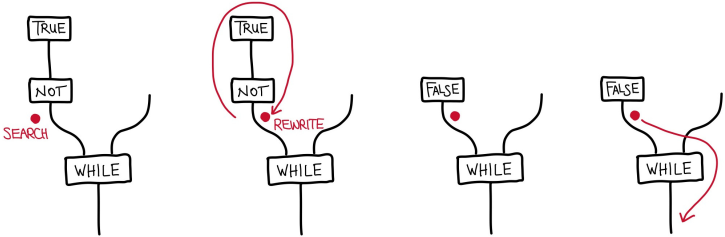
- Take D' to be the contour of O according to f.

From Syntax to Graph Rewriting Semantics[2]

- token = current "focus"
- token moves or triggers a rewrite



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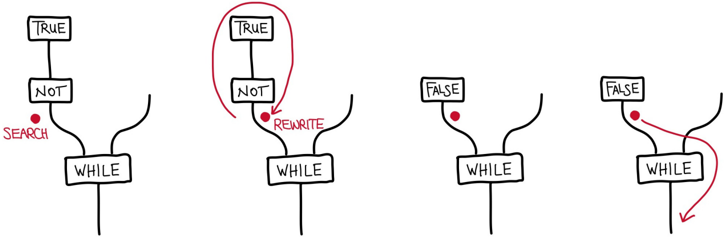

- objects in the contour: potential token positions
- morphisms in the contour: potential token movement

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Contours for Terms

- how does control flow translate from graphs to terms?

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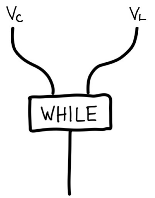

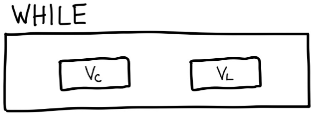

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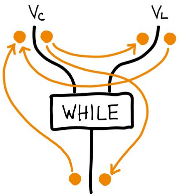

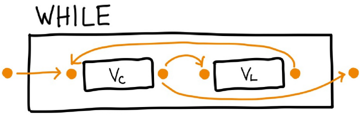

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Thoughts or questions? malin.altenmuller@huawei.com

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THANK YOU FOR YOUR ATTENTION!

References

Melliès, Paul-André and Noam Zeilberger. "Parsing as a lifting problem and the Chomsky-Schützenberger representation theorem". In: *MFPS 2022 - 38th conference on Mathematical Foundations for Programming Semantics*. Ithaca, NY, United States, July 2022. URL: https://hal.archives-ouvertes.fr/hal-03702762.

Muroya, Koko and Dan R. Ghica. "The Dynamic Geometry of Interaction Machine: A Token-Guided Graph Rewriter". In: *Logical Methods in Computer Science* Volume 15, Issue 4 (Oct. 2019). DOI: 10.23638/LMCS-15(4:7)2019. URL: https://lmcs.episciences.org/5882.

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Image CFG from:
Al-Ekram, R. & Kontogiannis, Kostas. (2004). Source code modularization using lattice of concept slices. 195- 203. 10.1109/CSMR.2004.1281420.